Closing Wed: HW_2A,2B
Closing Fri: HW_2C
My extra office hours today 1:15-3:00pm in Com. B-006 (next to MSC)

Quick review of foundations:

Definition of Definite Integral:

If $\Delta x = \frac{b-a}{n}$ and $x_i = a + i\Delta x$, then

$$\int_a^b f(x)\, dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$

= "signed" area between f(x) and the x-axis from x=a to x=b.

FTOC(1): Areas are antiderivatives!

$$\frac{d}{dx}\left(\int_a^x f(t)dt\right) = f(x)$$

FTOC(2): F(x) <u>any</u> antiderivative of f(x),

$$\int_a^b f(x)dx = F(b) - F(a)$$

Entry Task: Evaluate

$$\int_0^4 e^x + \sqrt{x^3}\, dx$$

$$\int_3^6 \frac{4}{x} - \frac{2}{x^2}\, dx$$

5.4 The Indefinite Integral and Net/Total Change

Def'n: The **indefinite integral** of $f(x)$ is defined to be the general antiderivative of $f(x)$.
And we write

$$\int f(x)dx = F(x) + C,$$

where $F(x)$ is any antiderivative of $f(x)$.

Net Change and Total Change

Assume an object is moving along a straight line (up/down or left/right).

set up:

Let s(t) = `location of object at time *t*'

v(t) = `velocity at time t'

positive v(t) means up/right

negative v(t) means down/left

The FTOC (part 2) says

$$\int_a^b v(t)dt = s(b) - s(a)$$

i.e. `integral of rate of change of dist.'

= `net change in distance'

5.5 Substitution - *Motivation*:

1. Find the following derivatives

Function	Derivative?
$\cos(x^2)$	
$\sin(x^4)$	
$\mathrm{e}^{\tan(x)}$	
$(\ln(\mathrm{x}))^3$	
$\ln(\mathrm{x}^4+1)$	

2. Rewrite each as integrals:

$$\int \qquad dx = \cos(x^2) + C$$

$$\int \qquad dx = \sin(x^4) + C$$

$$\int \qquad dx = \mathrm{e}^{\tan(x)} + C$$

$$\int \qquad dx = (\ln(\mathrm{x}))^3 + C$$

$$\int \qquad dx = \ln(\mathrm{x}^4 + 1) + C$$

3. Guess and check the answer to:

$$\int 7x^6 \sin(x^7)\, dx =$$

Observations:

1. We are reversing the “chain rule”.
2. In each case, we see

   “inside” = function inside another

   “outside” = derivative of inside

To help us mechanically see these connections, we use what we call:

The Substitution Rule:

If we write u = g(x) and du = g’(x) dx, then

$$\int f\big(g(x)\big)g'(x)dx = \int f(u)du$$